PROGRAM AND ABSTRACTS

Table of contents

International Workshop
on Hysteresis and Slow-Fast Systems

Program committee:
Pavel Gurevich (Free University of Berlin)
Anna Kuznetsova (Technical University of Berlin)
Dmitrii Rachinskii (University College Cork)
Sergey Tikhomirov (Free University of Berlin)

Organizing committee:
Pavel Gurevich (Free University of Berlin)
Yulia Jagodzinski (Technical University of Berlin)
Anna Kuznetsova (Technical University of Berlin)
Eyal Ron (Free University of Berlin)
Sergey Tikhomirov (Free University of Berlin)

Organized by:
Sonderforschungsbereich 910 / Collaborative Research Center 910:
Control of self-organizing nonlinear systems: Theoretical methods and concepts of application

General information

Dear participants,

Welcome to the International Workshop on Hysteresis and Slow-Fast Systems (HSFS-2011), December 12-14, 2011. The workshop is being organized by young scientists of SFB 910: Control of self-organizing nonlinear systems: Theoretical methods and concepts of application.

The workshop takes place at the conference center Leucorea, Lutherstadt Wittenberg, Germany.

The workshop is devoted to systems with hysteresis and their connection with slow-fast systems and related topics. The goal of the workshop is to bring together scientists working in the neighboring fields and stimulate active exchange of ideas and methods. The emphasis is put on:

- reaction-diffusion equations with hysteresis;
- systems with different temporal and spatial scales (slow-fast systems, homogenization);
- applications to systems with self-organization in biology, biophysics, and neurophysiology.

Email: hsfs2011@math.fu-berlin.de

Website: http://www.math.fu-berlin.de/hsfs2011/

Timetable

Monday, December 12

08:52 - 09:33		Train Berlin-Wittenberg
09:30 - 10:50		Registration
10:50 - 11:00		Opening ceremony
11:00 - 11:30	Pavel Krejčí	Rate independent hysteresis as a limit case of slow-fast systems
11:30 - 12:00	Thomas Erneux	Slow-fast asymptotics for delay differential equations
12:00 - 12:30	Martin Brokate	Optimal control of an ODE system coupled with the vectorial stop operator
12:30 - 14:00		Lunch
14:00 - 14:30	Jana Kopfová	A model from thermoplasticity with temperature-dependent Preisach hysteresis
14:30 - 15:00	Pavel Gurevich	Reaction-diffusion equations with spatially distributed hysteresis
15:00 - 15:30		Coffee break
15:30 - 16:00	Anna Marciniak-Czochra	Travelling waves behaviour in a hysteresis-based model of pattern formation
16:00 - 16:30	Daria Apushkinskaya	Parabolic obstacle type problems — a survey
16:30 - 17:00		Coffee break
17:00 - 19:00		*Informal discussions*
19:00 - 21:00		Dinner

Tuesday, December 13

08:00 - 09:00		Breakfast
09:00 - 09:30	Markus Bär	Where homogenization fails: Arrhythmias and percolation in a discrete model of cardiac tissue
09:30 - 10:00	Sergio Alonso	Effective medium theory for heterogeneous chemical reaction-diffusion media
10:00 - 10:30		Coffee break
10:30 - 11:00	Marita Thomas	Convergence of rate-independent processes
11:00 - 11:30	Dmitrii Rachinskii	An attempt at modelling memory effects in dynamics of spread of infectious disease
11:30 - 12:00		Coffee break
12:00 - 12:30	Christian Kuehn	Hunting french ducks in a noisy environment
12:30 - 13:00	Ilya Schurov	Canard cycles in generic slow-fast systems on the two-torus
13:00 - 14:30		Lunch
14:30 - 15:00	Michela Eleuteri	Determination of the equivalent anisotropy properties of polycrystalline magnetic materials: theoretical aspects and numerical analysis
15:00 - 15:30	Ondrej Hovorka	Rate-dependent hysteresis losses in ensembles of magnetic nanoparticle clusters
15:45 - 17:15		Excursion
17:30 - 19:00		*Informal discussions*
19:00 - 22:00		Banquet

Wednesday, December 14

08:00 - 09:00		Breakfast
09:00 - 09:30	Augusto Visintin	P.D.E.s with discontinuous hysteresis
09:30 - 10:00	Jens Starke	Equation-free analysis in neuroscience
10:00 - 10:30		Coffee break
10:30 - 11:00	Anna Kuznetsova	Controlling neural wave dynamics by nonlocal and time-delayed feedback
11:00 - 11:30	Phi Ha	On the stability of differential-algebraic PDEs by time-delayed feedback control
11:30 - 12:00		Coffee break
12:00 - 12:30	Thomas Roche	(Quasivariational) Sweeping processes on functions of bounded variation
12:30 - 13:00	Olaf Klein	Hysteresis operators for vector-valued inputs and their representation by functions on strings
13:00 - 13:10		Closing ceremony
13:10 - 14:30		Lunch
14:30 - 15:30		*Informal discussions and coffee*
16:22 - 17:05		Train Wittenberg-Berlin

Abstracts

Effective medium theory for heterogeneous chemical reaction-diffusion media

Sergio Alonso, Markus Bär
Physikalisch-Technische Bundesanstalt, Berlin, Germany

The small scale of the heterogeneities usually allows an effective description of the inhomogeneous system. Specific homogenization theories have been already suggested, but we propose a general effective medium theory based on the homogeneization of reaction-diffusion systems [1,2]. We consider a system where domains of phase 2 (heterogeneities) are randomly dispersed in a medium of phase 1. The reactivity and diffusion of the reactants take different values if they are inside or outside of such domains. If the heterogeneities are small, we can calculate effective values for the diffusion and the reactivity. Here, we validate the predictions of the effective medium theory by the numerical calculation of the velocity of a wave in reaction-diffusion systems under the presence of static obstacles. We compare the numerical results obtained for bistable [2] and excitable [3] media with the predictions of the effective homogeneous medium theory. Some of the results are applied to chemical waves in microemulsions.

[1] S. Alonso, R. Kapral and M. Bär, *Phys. Rev. Lett.* 102, 238302, 2009.
[2] S. Alonso, M. Bär and R. Kapral, *J. Chem. Phys.* 131, 214102, 2009.
[3] S. Alonso, J. Löber, M. Bär and H. Engel, *Eur. Phys. J. Special Topics*, 187, 31, 2010.

Parabolic obstacle type problems — a survey

Daria Apushkinskaya
Saarland University, Saarbrücken, Germany

In this talk we present a short survey on the special class of the parabolic free boundary problems, which is called the obstacle type problems. We discuss the different aspects of one- and two-phase obstacle type problems, such as the optimal regularity of solutions, the classification of free boundary points and the regularity of the free boundaries.

Where homogenization fails: Arrhythmias and percolation in a discrete model of cardiac tissue

Sergio Alonso, **Markus Bär**
Physikalisch-Technische Bundesanstalt, Berlin, Germany

Modelling of cardiac dynamics usually is carried out with spatially homogeneous reaction-diffusion type equation, that are obtained by assuming that the width of the electrical wave front in the tissue is much larger than the cardiac muscle cells. This homogenization approach fails often in damaged tissue where the coupling of the cells is substantially weakened and many cells may lose their excitatory properties. Then, discrete models representing individual cells and their (heterogeneous) excitation and coupling properties are appropriate to model the cellular response. Here, arrhythmias in cardiac tissue are related with electrical wave propagation in the heart. Cardiac tissue is modelled by a discrete network. It is shown by extensive simulations in a physiologically realistic discrete model that a wave crossing a heterogeneous region of cardiac tissue may breakup and produce irregular patterns, when the fraction of heterogeneites is approaching the percolation threshold of the cell network. The results are generic for heterogeneous excitable media and do not depend on the exact structure of the cells in cardiac tissue.

Optimal control of an ODE system coupled with the vectorial stop operator

Martin Brokate
Technical University of Munich, Munich, Germany

We consider an optimal control problem for a system of ordinary differential equations coupled to the evolution variational inequality which represents the vectorial stop hysteresis operator. We prove necessary optimality conditions of first order and study the regularity of the adjoint system.

Determination of the equivalent anisotropy properties of polycrystalline magnetic materials: theoretical aspects and numerical analysis

M. Eleuteri
Universit di Verona, Verona, Italy

O. Bottauscio
Istituto Nazionale di Ricerca Metrologica, Turin, Italy

V. Chiadò Piat
Politecnico di Torino, Turin, Italy

L. Lussardi
Universit‘ Cattolica del Sacro Cuore, Brescia, Italy

A. Manzin
Istituto Nazionale di Ricerca Metrologica, Turin, Italy

The aim of this talk is the determination of the equivalent anisotropy properties of polycrystalline magnetic materials, modeled as an assembly of monocrystalline grains with a stochastic spatial distribution of easy axes. The theory of Γ-convergence is here adopted to homogenize the anisotropic contribution in the energy functional and derive the equivalent anisotropy properties. The reliability of this approach is investigated focusing either on the micromagnetic computation of reversal processes and on the computation of the static hysteresis loops of polycrystalline magnetic thin films, starting from the numerical integration of the Landau-Lifshitz-Gilbert equation.

Slow-fast asymptotics for delay differential equations

Thomas Erneux, Lionel Weicker
Université Libre de Bruxelles, Brussels, Belgium

Laurent Larger
University of Franche-Comté, Besançon, France

Time-delayed systems are known to exhibit symmetric square-waves oscillating with a period close to twice the delay. Here, we show that strongly asymmetric square-wave oscillations of period close to one delay are possible. The plateau lengths of the square-wave can be tuned by changing a control parameter. The problem is investigated both experimentally and numerically by studying the response of a simple optoelectronic oscillator. An asymptotic approximation of the square-wave periodic solution based on the large value of the delay allows an analytical understanding of the bifurcation diagram. Mathematically, the construction of the solution relies on a slow-fast orbit in a phase plane exhibiting a S-shaped slow manifold. The analysis is however different from the standard construction of limit-cycles of slow-fast Linénard systems because of an extra periodicity condition.

Reaction-diffusion equations with spatially distributed hysteresis

Pavel Gurevich, Sergey Tikhomirov
Free University of Berlin, Berlin, Germany

Roman Shamin
Shirshov Institute of Oceanology of the Russian Academy of Sciences, Moscow, Russia

We consider reaction-diffusion equations involving a hysteretic discontinuity which is defined at each spatial point. In particular, such problems describe chemical reactions and biological processes in which diffusive and nondiffusive substances interact according to hysteresis law.

Hysteresis may switch at different spatial points at different time moments, dividing the spatial domain into subdomains where hysteresis has the same state and thus defining spatial topology of hysteresis. The boundaries between the subdomains are free boundaries whose motion depends both on the reaction-diffusion equation and hysteresis. The interplay of those two leads to formation of spatio-temporal patterns.

We formulate a theorem that states that the problem has a unique solution as long as this solution preserves spatial topology of hysteresis. We discuss mechanisms of changing the topology. The most important mechanism will be related to spatial nontransversality of the solution.

Supported by DFG through SFB 910.

On the stability of differential-algebraic PDEs by time-delayed feedback control

Volker Mehrmann, **Phi Ha**
Technical University of Berlin, Berlin, Germany

Self-organizing control of dynamic chaos is a very interesting effect which has been intensively studied during the last two decades. It has been demonstrated using a standard method of delay coordinated that a large number of distinct unstable periodic orbits on a chaotic attractor can be obtained from one input signal. One of the main topics which currently attracts lots of interest is to stabilize a desire, periodic unstable orbit using a certain type of delayed feedback control. In this talk, we discuss that question for the limiting system of singularly perturbed coupled PDE

$$\begin{aligned} u_t &= \Delta_x u + g(u,v),\ t > 0, \\ \epsilon v_t &= f(u,v), \end{aligned}$$

where $0 \leq \epsilon \ll 1$ is a singular perturbation parameter.

Rate-dependent hysteresis losses in ensembles of magnetic nanoparticle clusters

O. Hovorka, R. F. L. Evans, R. W. Chantrell
The University of York, York, United Kingdom
G. Friedman
Drexel University, Philadelphia, PA, United States

Hysteresis is ubiquitous in magnetic nanoparticle systems and understanding how it emerges from complex interactions and for different time scales is a long-standing issue in magnetism research. Understanding the phenomenon is most important for engineering magnetic nanoparticle structures of well-controlled properties in magnetic recording, hysteresis loss optimization in hyperthermia cancer treatment in biomedicine, or biological and chemical sensing, to name a few examples.

In this work we address one of the general questions related to the influence of thermal activation processes on hysteresis loss. Employing large-scale computational modeling based on the master-equation framework we investigate the influence of dipolar interactions on thermal hysteresis loops in ensembles of magnetic nanoparticle chains and clusters. We show that the directional dependence of dipolar interactions results in enhanced or reduced hysteresis loss, depending on the distribution of particles' anisotropy axes and particle chain orientations with respect to the external field. Additional hysteresis loss reduction occurs in case of particle clusters due to possibility of the frustration phenomenon not present for topologically simpler chains.

Hysteresis operators for vector-valued inputs and their representation by functions on strings

Olaf Klein
Weierstrass Institute for Applied Analysis and Stochastics, Berlin, Germany

In [1], Brokate and Sprekels have derived representation formulas for hysteresis operators acting on scalar-valued continuous input functions being piecewise monotone. In the current work, hysteresis operators dealing with inputs in a general normed vector space are investigated, and a similar representation formulae is derived.

Let $T > 0$ denote some final time. Let $(V, \|\cdot\|)$ be some normed vector space and let Y be some nonempty set. Following the monographs [1,2,5], an operator $\mathcal{H} : D(\mathcal{H})(\subseteq \mathrm{Map}([0,T];V)) \to \mathrm{Map}([0,T];Y)$ with $D(\mathcal{H}) \neq \emptyset$ is denoted as *hysteresis operator* if it is rate-independent and causal.

To define an appropriate generalization of monotonicity for scalar function for functions with values in V, we consider the composition of a **monoto**ne with an **affine** function, ending up with a **monotaffine** function according to the following definition:

1. Let some $t_1, t_2 \in [0,T]$ with $t_1 < t_2$ and some function $u : [0,T] \to V$ be given. Then u is denoted as *monotaffine on* $[t_1, t_2]$ if there exists some increasing function $\beta : [t_1, t_2] \to [0,1]$ such that

$$u(t) = (1 - \beta(t))u(t_1) + \beta(t)u(t_2), \quad \forall t \in [t_1, t_2]. \tag{1}$$

2. A function $u : [0,T] \to V$ is denoted as *piecewise monotaffine* if there exists a decomposition $0 = t_0 < t_1 < \cdots < t_N = T$ of $[0,T]$ such that for $1 \leq i \leq N$ u is monotaffine on $[t_{i-1}, t_i]$. Let $C_{\mathrm{p.m.a.}}([0,T];V)$ be the subset of all piecewise monotaffine functions in $C([0,T];V)$.

3. Let $u \in C_{\mathrm{p.m.a.}}([0,T];V)$ be given. The *standard monotaffinicity partition of* $[0,T]$ *for* u is the uniquely defined decomposition $0 = t_0 < t_1 < \cdots < t_N = T$ of $[0,T]$ such that for $1 \leq i \leq N$ t_i is the maximal number in $]t_{i-1}, T]$ such that u is monotaffine on $[t_{i-1}, t_i]$.

As for the scalar case in [1], we can consider strings of elements of V:

1. Let $(v_0, \ldots, v_N) \in V^{N+1}$ be given. Then $(v_0, \ldots, v_N)$ is denoted as *convexity triple free string,* if for all $1 \leq i < N$ and all $\lambda \in [0,1]$ it holds that $v_i \neq \lambda v_{i-1} + (1-\lambda)v_{i+1}$.

2. Let $S_F(V)$ be the set of all convexity triple free strings of elements of V.

The representation result for hysteresis operators with scalar inputs as in Prop. 2.2.5 in [1] is extended to hysteresis operators with inputs in normed space. Details can be found in the forthcoming papers [3,4].

1. For **every hysteresis operator** $\mathcal{G} : C_{\textbf{p.m.a.}}([0,T];V) \to \textbf{Map}([0,T];Y)$ there **exists a unique function** $G : S_F(V) \to Y$ such that for all $u \in C_{\text{p.m.a.}}([0,T];V)$ and the corresponding standard monotaffinicity decomposition $0 = t_0 < t_1 < \cdots < t_N = T$ it holds that

$$\mathcal{G}[u](t) = G\left(u(t_0), u(t)\right), \qquad \forall\, t \in [t_0, t_1], \tag{2}$$

$$\mathcal{G}[u](t) = G\left(u(t_0), \ldots, u(t_{i-1}), u(t)\right), \qquad \forall\, t \in]t_{i-1}, t_i],\ 2 \le i \le N. \tag{3}$$

2. Every function $G : S_F(V) \to Y$ generates a hysteresis operator $\mathcal{G} : C_{\text{p.m.a.}}([0,T];V) \to \text{Map}([0,T];Y)$ by the method presented above.

[1] M.Brokate, J. Sprekels,“ Hysteresis and Phase Transitions”, *Springer-Verlag*, New York, 1996

[2] P. Krejčí, “Hysteresis, Convexity and Dissipation in Hyperbolic Equations”, volume 8 of *Gakuto Int. Series Math. Sci. & Appl.*, *Gakkōtosho*, Tokyo, 1996

[3] O. Klein, “Representation of hysteresis operators for vector-valued continuous monotaffine input functions by functions on strings”, in preparation

[4] O. Klein, “Representation of hysteresis operators for vector-valued inputs by functions on strings”, *Physica B*, in Press. `http://dx.doi.org/10.1016/j.physb.2011.10.015`

[5] A. Visintin, “Differential Models of Hysteresis”, *Springer-Verlag*, New York, 1994.

A Model from thermoplasticity with temperature-dependent Preisach hysteresis

Jana Kopfová

Mathematical Institute of the Silesian University, Opava, Czech Republic

Classical models for shape memory materials ([1,2,5]) can be alternatively described by a constitutive equation involving hysteresis operators.

To account for the memory erasure during the austenite-martensite phase transition, we propose here a new model: The stress-strain law we assume to be given by a Preisach-like formula

$$\sigma = E\varepsilon - \int_0^\infty g(r, \eta_r, \theta)\mathrm{d}r \tag{1}$$

with a given constitutive function g, where σ is the stress, ε is the strain, θ is the absolute temperature and η_r is the solution of the variational inequality (for given ε) of the following form:

$$\begin{aligned} &|\varepsilon(t) - \eta_r(t)| \le r \quad \forall t \in [t_1, t_2]; \\ &(\mu_1(\theta)\dot{\eta}_r(t) + \mu_2(\theta)(\eta_r(t) - \varepsilon(t)))(\varepsilon(t) - \eta_r(t) - y) \ge 0 \quad \text{a.e. } \forall |y| \le r\,, \end{aligned} \tag{2}$$

with given functions μ_1, μ_2 such that μ_1 vanishes for high temperatures (i.e. no memory) and μ_2 vanishes for low temperatures (i.e. the standard play operator).

The obvious drawback of this definition is its relative complexity. On the other hand, we showed in [4] that the model is thermodynamically consistent, and in [3] that the full system of dynamical balance equations is well posed.

[1] F. Auricchio and L. Petrini, "A three-dimensional model describing stress-temperature induced solid phase transformations. Part II: thermomechanical coupling and hybrid composite applications," *Internat. J. Numer. Methods Engrg.*, vol. 61, pp. 716–737, 2004.

[2] F. Auricchio, R.L. Taylor, and J. Lubliner,"Shape-memory alloys: macromodelling and numerical simulations of the superelastic behavior," *Comput. Methods Appl. Mech. Engrg.*, vol. 146, pp. 281–312, 1997.

[3] M. Eleuteri, J. Kopfová and P.Krejčí, A model from thermoplasticity with temperature- dependent Preisach hysteresis, in preparation

[4] J. Kopfová and P.Krejčí "A Preisach type model for temperature driven hysteresis memory erasure in shape memory materials", *Cont. Mech. Thermodynam.* vol. 23 , pp. 125–137, 2011.

[5] A. C. Souza, E. N. Mamiya, and N. Zouain, "Three-dimensional model for solids undergoing stress-induced phase transformations," *European J. Mech. A Solids*, vol. 17, pp. 789–806, 1998.

Rate independent hysteresis as a limit case of slow-fast systems

Pavel Krejčí
Institute of Mathematics, Czech Academy of Sciences,
Prague, Czech Republic

A singular evolution differential equation

$$\alpha \dot{x}(t) = \Phi(u(t), x(t))$$

with a non-monotone equilibrium set $\Phi(u, x) = 0$, a given input $u(t)$, and with a small coefficient α in front of the time derivative, is known to exhibit a limit hysteresis behavior as the singular parameter α tends to zero. Following [4,5], we investigate possible choices of the function space and of the topology in which the convergence takes place. One alternative consists in choosing the space of left-continuous regulated functions (that is, functions that admit a finite right limit at each point of their domain of definition, see [1]), endowed with the so-called BO-topology ([2]) related to the generalized Helly selection principle for regulated functions established by Dana Fraňková in [3]. The limit may not be unique if no additional regularity is assumed either for the

input or for the equilibrium set.

[1] G. Aumann,"Reelle Funktionen". Springer-Verlag, Berlin-Göttingen-Heidelberg, 1954 (In German).

[2] M. Brokate, P. Krejčí, "Duality in the space of regulated functions and the play operator". *Math. Z.* 245, 667–688, 2003.

[3] D. Fraňková, "Regulated functions". *Math. Bohem.*, 119, 20–59, 1991.

[4] P. Krejčí, "Hysteresis in singularly perturbed problems". *Singular Perturbation and Hysteresis*, Eds. M.P. Mortell, R.E. O'Malley, A. Pokrovskii, V. Sobolev, SIAM, 73–100, 2005.

[5] P. Krejčí, "The hysteresis limit in relaxation oscillation problems". *International Workshop on Hysteresis and Multi-scale Asymptotics*, Eds. M.P. Mortell, R.E. O'Malley, A. Pokrovskii, V. Sobolev, *Journal of Physics: Conference Series*, 22, 103–123, 2005.

Hunting french ducks in a noisy environment

Christian Kuehn
Technical University of Vienna, Austria

Abstract: We study the influence of noise on oscillations in multiple time scale systems. Our results form a main building block to understand mixed-mode oscillations that appear in many different applications. The normal form of a folded node singularity in three dimensions with two slow variables and one fast variable will be introduced which produces canard ("duck") orbits generating local oscillations. Key technique are the analysis of variational equations and concentration estimates for meta-stable sample paths.

Joint work with N. Berglund (Orleans) and B. Gentz (Bielefeld).

Controlling neural wave dynamics by nonlocal and time-delayed feedback

Anna Y. Kuznetsova, Eckehard Schöll, Markus A. Dahlem
Institute for Theoretical Physics of Berlin Technical University,
Berlin, Germany

We model the propagation of traveling waves of spreading depression (cellular depolarization) in the cortex. An invasion of the healthy state by the depolarized state can be modeled by reaction-diffusion systems with one or two variables. The first variable is an activator, a lump variable, representing inward ionic currents and the extracellular potassium concentration, and the second variable is an inhibitor representing outward ionic currents. We use the FitzHugh-Nagumo equations with diffusion. The inhibitor dynamics changes on a slower time scale (time scale separation parameter $\varepsilon \ll 1$

is small). By adding a feedback control term, we can modulate the spatio-temporal dynamics. First, we consider the fast variable only ($\varepsilon = 0$) and modulation of the front propagation speed by spatially nonlocal and time-delayed feedback control. Variation of control parameters (feedback gain, spatial or temporal control scales) slows down or speeds up the front propagation. Next, we consider fast and slow variables together (small but finite ε) and study the pulse propagation. The control planes for time-delayed and spatially nonlocal feedback control are similar in the efficiency of transient wave suppression except for large time delays. Bifurcation diagrams as function of ε are considered. The control can shift the domain of the regime of traveling waves to smaller or larger values of ε. Thus, the control in the brain may target the slowness of inhibitor dynamics or rapidity of activator dynamics to stop spreading depression waves. Recent experimental data direct this modeling approach opening new perspective for stroke, traumatic brain injury, and migraine therapy.

Supported by DFG through SFB 910.

Travelling waves behaviour in a hysteresis-based model of pattern formation

Anna Marciniak-Czochra
University of Heidelberg, Heidelberg, Germany

Coupling diffusion process of signalling molecules with nonlinear dynamics of intracellular processes and cellular growth and transformation leads to reaction-diffusion-ODEs models, which differ from the usual reaction-diffusion systems. One of the mechanisms of pattern formation in such systems is based on the existence of multiple steady states and hysteresis in the ODEs subsystem. Diffusion tries to average different states and is the cause of spatio-temporal patterns. To investigate mechanism of pattern formation based on these concepts we propose a basic model consisting of a reaction-diffusion equation coupled with one ordinary differential equation. In such model infinitely many discontinuous stationary solutions can be constructed. We show that in some regime of parameters the dynamics of the model is governed by a travelling wave phenomenon. As shown in numerical simulations existence of travelling waves excludes formation of stable patterns and the solutions converge to spatially homogenous states. In the case when travelling waves do not exist, we observe formation of stable spatially heterogenous patterns. The model is motivated by biological applications and elucidates a possible mechanism of symmetry breaking and pattern formation. We discuss these issues on the example of head formation in Hydra, which is a model organism of developmental biology.

The talk is based on a joint research with Alexandra Köthe (University of Heidelberg) and Izumi Takagi (Tohoku University).

An attempt at modelling memory effects in dynamics of spread of infectious disease

Dmitrii Rachinskii
Department of Applied Mathematics, University College Cork, Cork, Ireland

Modification of behaviour in response to changes in the environment or ambient conditions, based on memory, is typical of the human and possibly many animal species. An obvious example of such adaptive behaviour is, for instance, switching to a safer behaviour when in danger, from either a predator or an infectious disease. In human society such switching to safe behaviour is particularly apparent during epidemics. Mathematically, changes of behaviour in response to changes in the conditions can be described by models involving switching. In most cases, this switching is assumed to depend on the system state, and thus it disregards the history and, therefore, memory.

Here we explore a possibility to introduce memory into a population dynamics model using the Preisach hysteresis operator. We illustrate the idea, using a simple SIR compartmental model that is applicable in epidemiology. Our goal is to show why and how hysteresis can arise in such a model, and how it may be applied to describe memory effects. We present a case study based on numerical simulations, where system stabilizes at a set of equilibrium states and present a simple analysis of this set. We then discuss possible extensions of this modelling approach.

(Quasivariational) Sweeping processes on functions of bounded variation

Thomas Roche
Technical University of Munich, Munich, Germany

We study the sweeping process in Hilbert spaces. Using a method due to Krejčí and Laurençot to extend the sweeping process to BV we consider quasivariational sweeping processes on functions of bounded variation. We prove existence and uniqueness of a solution. Part of the results we present have been obtained in joint work with Pavel Krejčí.

Canard cycles in generic slow-fast systems on the two-torus

Ilya V. Schurov
National Research University "Higher School of Economics", Moscow, Russia

Generic slow-fast systems with only one (time-scaling) parameter on the two-torus have attracting canard cycles for arbitrary small values of this parameter. This is in drastic contrast with the planar case, where canards usually occur in two-parametric families. In our talk, general case of nonconvex slow curve with several fold points is considered. We show that the number of canard cycles in such systems can be effectively computed and is no more than the number of fold points. This estimate is sharp for every system from some explicitly constructed open set.

Equation-free analysis in neuroscience

Jens Starke
Technical University of Denmark, Kongens Lyngby, Denmark

For cases where for microscopically defined models no explicit equations are known for the behaviour on the macroscopic scale, equation-free techniques provide a numerical technique for systematic investigations of the macroscopic properties. As example in neuroscience, the neural network in the olfactory bulb is investigated. The macroscopic neural network activities and their dependence on biological parameters such as kinetic parameters or the network topology is investigated using equation-free continuation methods and bifurcation analysis. A spike and response model is used as microscopic model for the network in the olfactory bulb, consisting of mitral cells which are coupled in an inhibitory way via granular cells. The input of this network is well defined by odor evoked stimulus specific spatio-temporal patterns on a so-called glomeruli level. This is taken from in vivo experimental data obtained by optical imaging techniques. A spatial independent component analysis of this high-resolution imaging data was used to identify and separate different neuronal populations based on their stimulus specific spatio-temporal activation. The investigated topology dependent phenomena of spatio-temporal pattern formation include contrast enhancement between several spatially close activations and hysteresis effects in odorant discrimination between similar odorants depending on the concentration ratios of odorant mixtures.

An outlook will be given about systematic investigations in situations where no accurate mathematical models are available but where it is possible to perform a bifurcation analysis for experiments. This will be demonstrated for an experiment in mechanical engineering and possible future applications in neuroscience experiments will be discussed.

This is in parts joint work with E. Bureau, C. Ellsaesser, A. Grinvald, J. Midtgaard, D. Omer, J. Reidl, I. Santos, F. Schilder, H. Spors, J. Thomsen.

Convergence of rate-independent processes

Marita Thomas
Weierstrass Institute for Applied Analysis and Stochastics, Berlin, Germany

This contribution focuses on rate-independent evolutionary processes in solids. In the framework of generalized standard materials processes like plastic deformation, damage or fracture in a solid are modelled with the aid of an internal variable, which is incorporated to the constitutive law in order to describe the changes of the material's elastic behavior due to this evolving process. The mathematical modelling of such processes is done using the so-called energetic formulation, which is solely based on an energy functional and a dissipation potential. The first comprises the stored energy and the amount of energy generated by the external loadings. The latter accounts for the evolution of the internal variable.

The aim of this contribution is to present an abstract theorem on the convergence of a sequence of rate-independent processes to a limit process, which was developed by A. Mielke, T. Roubíček and U. Stefanelli in 2008. It applies to convergence problems arising due to e.g. regularization or dimension reduction. Moreover it plays a role in homogenization procedures. Its application will be discussed in examples from hysteresis, damage and fracture.

P.D.E.s with discontinuous hysteresis

Augusto Visintin
University of Trento, Trento, Italy

A large number of phenomena in physical and nonphysical sciences exhibit hysteresis. By now it is widely recognized that hysteresis may be represented as *rate-independent memory,* although in applications rate-independence may be disguised by the overlapping of rate-dependence due to viscosity or to the coupling with rate-dependent P.D.E.s.

The notion of hysteresis operator was introduced and thoroughly investigated by the Russian School of Krasnosel'skiĭ and Pokrovskiĭ [1], who in the 1970's pioneered the notion of continuous hysteresis operator. Discontinuous hysteresis was first represented by a system of variational inequalities in 1982 in [2] (see also the more easily available [3]).

These operators were then coupled with P.D.E.s, see e.g. the monographs [3,4,5]. An alternative representation of rate-independent evolution, that became known as the *energetic approach,* was introduced at the end of the 1990's by Mielke and others, see e.g. [6]. This talk will briefly review these achievements.

A weak formulation in Sobolev spaces will then be provided for the Cauchy problem for a scalar quasilinear hyperbolic equation with discontinuous hysteresis:

$$\frac{\partial^2}{\partial t^2}[u+\mathcal{F}(u)]-\Delta u=f \qquad \text{with } \mathcal{F} \text{ hysteresis operator.}$$

For example, this arises in the study of the Maxwell system accounting for displacement current, see [7]. Existence of a solution will be briefly illustrated.

The corresponding vector problem will then be addressed for $\mathcal{F}$ equal to a *vector relay operator*, see [8]. The extension to the (vector) Preisach model will be discussed, and a counterexample will be illustrated. The occurrence of a mesoscopic structure will be discussed, and an alternative two-scale model will then be outlined.

[1] M.A. Krasnosel'skiĭ, A.V. Pokrovskiĭ, "Systems with Hysteresis", *Springer, Berlin*, 1989 (Russian ed. Nauka, Moscow 1983)

[2] A. V., "A phase transition problem with delay", Control and Cybernetics, 11, 5–18, 1982

[3] A. V.,"Differential Models of Hysteresis", *Springer, Berlin*, 1994

[4] M. Brokate, J. Sprekels, "Hysteresis and Phase Transitions", *Springer, Berlin*, 1996

[5] P. Krejčí,"Convexity, Hysteresis and Dissipation in Hyperbolic Equations", *Gakkotosho, Tokyo*, 1997

[6] A. Mielke, F. Theil, "A mathematical model for rate-independent phase transformations with hysteresis", In H.-D. Alber, R. Balean, and R. Farwig, editors, *Proceedings of the Workshop on Models of Continuum Mechanics in Analysis and Engineering*, 117-129. *Shaker-Verlag*, 1999

[7] A. V., "Quasi-linear hyperbolic equations with hysteresis", *Ann. Inst. H. Poincaré. Nonlinear Analysis*, 19, 451-476, 2002

[8] A. V., "Maxwell's equations with vector hysteresis", *Archive Rat. Mech. Anal.*, 175, 1–38, 2005

Excursion and restaurant

Excursion

An excursion around Lutherstadt Wittenberg takes place on

Tuesday, December 13, 15:45.

We meet in front of Leucorea.

Restaurant

A banquet is organized on

Tuesday, December 13, 19:00

in the restaurant “Haus des Handwerks”, Collegienstraße 53a.

List of participants

Alonso, Sergio	Physikalisch-Technische Bundesanstalt, Berlin, Germany
Apushkinskaya, Daria	Saarland University, Saarbrücken, Germany
Azouani, Abderrahim	Free University of Berlin, Berlin, Germany
Bär, Markus	Physikalisch-Technische Bundesanstalt, Berlin, Germany
Brokate, Martin	Technical University of Munich, Munich, Germany
Darovskaya, Ksenia	Peoples' Friendship University of Russia, Moscow, Russia
Eleuteri, Michela	Verona University, Verona, Italy
Erneux, Thomas	Université Libre de Bruxelles, Brussels, Belgium
Gornostaeva, Elena	Technical University of Munich, Munich, Germany
Gurevich, Pavel	Free University of Berlin, Berlin, Germany
Ha, Phi	Technical University of Berlin, Berlin, Germany
Hovorka, Ondrey	University of York, York, United Kingdom
Kalinichenko, Dina	Saint-Petersburg State University, Saint-Petersburg, Russia
Karnauhova, Anna	Free University of Berlin, Berlin, Germany
Klein, Olaf	Weierstrass Institute for Applied Analysis and Stochastics, Berlin, Germany

Kopfova, Jana	Silesian University in Opava, Opava, Czech Republic
Krejčí, Pavel	Czech Academy of Sciences, Prague, Czech Republic
Kuehn, Christian	Technical University of Vienna, Vienna, Austria
Kuznetsova, Anna	Technical University of Berlin, Berlin, Germany
Liebscher, Stefan	Free University of Berlin, Berlin, Germany
Marciniak-Czochra, Anna	University of Heidelberg, Heidelberg, Germany
Rachinskii, Dmitrii	University College Cork, Cork, Ireland
Roche, Thomas	Technical University of Munich, Munich, Germany
Ron, Eyal	Free University of Berlin, Berlin, Germany
Schurov, Ilya	National Research University ‘‘Higher School of Economics’’, Moscow, Russia
Shamin, Roman	Shirshov Institute of Oceanology of the Russian Academy of Sciences, Moscow, Russia
Starke, Jens	Technical University of Denmark, Kongens Lyngby, Denmark
Thomas, Marita	Weierstrass Institute for Applied Analysis and Stochastics, Berlin, Germany
Tikhomirov, Sergey	Free University of Berlin, Berlin, Germany
Trageser, Vincent	Free University of Berlin, Berlin, Germany
Visintin, Augusto	University of Trento, Trento, Italy