Corrigendum to our paper
"Narrow operators and rich subspaces of Banach spaces with the Daugavet property"

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There is a gap in the proof of Lemma 3.10(b). This lemma reads as follows; we denote by $B(X)$ the unit ball of a Banach space X and by $S(X)$ its unit sphere.

Lemma 3.10 *Let T be a narrow operator on X.*

(a) *Let $S_1, \dots, S_n$ be a finite collection of slices and $U \subset B(X)$ be a convex combination of these slices, i.e., there are $\lambda_k \geq 0$, $k = 1, \dots, n$, $\sum_{k=1}^n \lambda_k = 1$, such that $\lambda_1 S_1 + \dots + \lambda_n S_n = U$. Then for every $\varepsilon > 0$, every $x_1 \in S(X)$ and every $w \in U$ there exists an element $u \in U$ such that $\|u + x_1\| > 2 - \varepsilon$ and $\|T(w - u)\| < \varepsilon$.*

(b) *The same conclusion is true if U is a relatively weakly open set.*

The proof of part (b) in the paper simply says, "This follows from (a) since given $w \in U$ there is a convex combination V of slices such that $w \in V \subset U$." (We have taken the liberty to correct a typo in the quote, and of course U is tacitly assumed to be nonempty.) It is true – see the references [8, Lemma II.1] or [21] cited in the paper – that there is such a V inside U; it is not clear, however, that V can be chosen to contain w.

We now wish to give a complete proof of (b). Let $\mathscr{W}$ be the family of all those convex combinations V of slices of $B(X)$ such that $V \subset U$ and let W be its union, i.e., $W = \bigcup \mathscr{W}$. We note that $\mathscr{W} \neq \emptyset$ by the references above and that $W \subset U$. Further, W is convex [if $0 < \lambda < 1$ and if x (resp. y) belongs to the convex combination of slices V_x (resp. V_y), then $\lambda x + (1 - \lambda) y \in \lambda V_x + (1 - \lambda) V_y$, which is a convex combination of slices], and it is weakly dense in U [as every nonvoid relatively weakly open subset of U encompasses a member of $\mathscr{W}$]. Since the norm closure $\overline{W}$ and the weak closure of the convex set W coincide and thus $U \subset \overline{W}$, there is an element $w' \in W$ such that $\|w - w'\| < \varepsilon' := \varepsilon/(1 + \|T\|)$. This w' belongs to some convex combination $V_{w'} \in \mathscr{W}$ of slices. Now apply part (a) to $V_{w'}$, w' and ε' to obtain some $u \in V_{w'} \subset U$; this u will work.